

Stationary light pulses in ultra cold atomic gasses

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We present a theoretical treatment of electromagnetically induced transparency and light storage using standing wave coupling fields in a medium comprised of stationary atoms, such as an ultra cold atomic gas or a solid state medium. We show that it is possible to create stationary pulses of light which have a qualitatively different behavior than in the case of a thermal gas medium, offering greater potential for quantum information processing applications.

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Electromagnetically induced transparency (EIT) [1] in ensembles of Λ atoms has been extensively studied, both experimentally [2, 3, 4, 5, 6, 7, 8] and theoretically [9, 10, 11, 12] as a means for coherent transfer of quantum states between photons and atoms. Although the feasibility of using such systems for an efficient quantum memory has been demonstrated, the nontrivial manipulation of, and interaction between, stored quantum states remains a challenge [13] due to the weak interactions between the atoms. Furthermore, the absence of any photonic component excludes the use of enhanced nonlinear optical interactions [14, 15] between stored pulses. Several schemes for using enhanced nonlinear optical interactions between slowly propagating, weak light pulses have been proposed [16, 17, 18], but the efficiency is limited since the reduction in group velocity is accompanied by a reduction in the energy of the pulse. Recent experimental progress in the coherent control of light pulses in atomic media [19] has demonstrated the possibility of generating stationary light pulses by using EIT with a standing wave coupling laser. It has been proposed [20] that such a system can be used for efficient nonlinear optical interactions between trapped pulses. A theoretical treatment of stationary light pulses in a thermal gas medium has been given in [21], but this theory does not apply to media comprised of stationary atoms, such as an ultra cold gas or a solid state crystal. In this Letter, we present a theoretical treatment of stationary light pulses in the more complicated case of a medium comprised of stationary atoms. It will be shown that stationary light pulses trapped in such media display a qualitatively different behavior and has greater potential for the kind of nonlinear optical interactions envisaged in [20] than in the thermal gas case.

We consider a medium of length L consisting of an ensemble of stationary three-level atoms in the Λ configuration (see inset in Fig. 1) interacting with a weak quantum field, called the probe field, and a strong classical field, called the coupling field, propagating parallel to the z axis. The probe field is resonant with the transition between the ground state $|b\rangle$ and the excited state $|a\rangle$, while the coupling field is resonant with the tran-

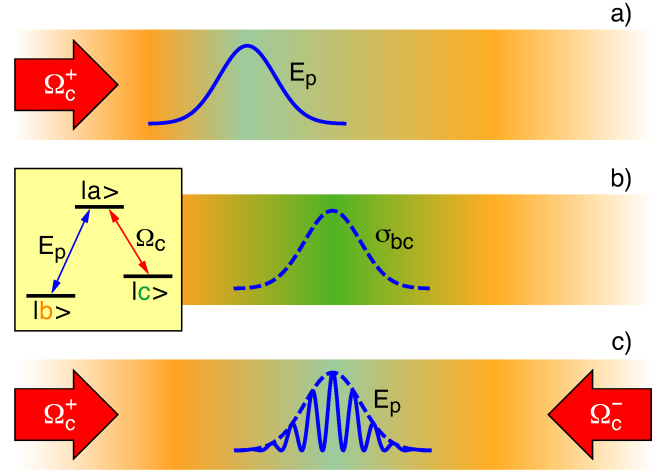


FIG. 1: (color online) Creation of a stationary probe pulse: (a) The probe pulse propagates into the medium under EIT conditions created by a traveling wave coupling laser. (b) The probe pulse is stored in the medium by switching off the coupling laser. (c) A stationary probe pulse is generated inside the medium by switching on a standing wave coupling laser. Inset: Energy level diagram for the three-level Λ atom.

sition between the metastable state $|c\rangle$ and the excited state. The two lower states of the Λ atoms are assumed to be nearly degenerate, such that the wave vectors of the probe and coupling fields are of equal magnitude.

The probe field operator is written in terms of slowly varying operators as

$$\hat{\mathbf{E}}_p(z, t) = \sqrt{\frac{\hbar\omega_p}{2\varepsilon_0 V}} \mathbf{e}_p E_p(z, t) e^{-i\omega_p t} + \text{h.c.} \quad (1)$$

where ω_p is the carrier frequency of the probe field, V is the quantization volume and \mathbf{e}_p is the polarization vector. Since we are considering standing wave fields, the operator E_p is written as a superposition of two traveling wave fields, denoted by E_p^\pm , propagating in opposite directions

$$E_p(z, t) = E_p^+(z, t) e^{ikz} + E_p^-(z, t) e^{-ikz}. \quad (2)$$

The Λ atoms are described by collective atomic oper-

ators, defined by

$$\hat{\sigma}_{\mu\nu}(z, t) = \frac{1}{N_z} \sum_{j=1}^{N_z} \hat{\sigma}_{\mu\nu}^j(t), \quad (3)$$

where N_z is the number of atoms in a small volume centered around z and $\hat{\sigma}_{\mu\nu}^j = |\mu_j\rangle\langle\nu_j|$. We introduce temporally slowly varying operators for the coherences

$$\hat{\sigma}_{ba}(z, t) = \sigma_{ba}(z, t)e^{-i\omega_p t}, \quad (4a)$$

$$\hat{\sigma}_{ca}(z, t) = \sigma_{ca}(z, t)e^{-i\omega_c t}, \quad (4b)$$

$$\hat{\sigma}_{bc}(z, t) = \sigma_{bc}(z, t)e^{-i(\omega_p - \omega_c)t}, \quad (4c)$$

where ω_p and ω_c are the carrier frequencies of the probe and coupling lasers, respectively. The evolution of the collective atomic operators is determined by Heisenberg-Langevin equations. Assuming that the probe field is weak compared to the coupling field, the Heisenberg-Langevin equations can be solved perturbatively in the probe field. To first order, the relevant Heisenberg-Langevin equations are

$$\dot{\sigma}_{ba} = i(g_p E_p + \Omega_c \sigma_{bc}) - \Gamma_{ba} \sigma_{ba} \quad (5a)$$

$$\dot{\sigma}_{bc} = i\Omega_c^* \sigma_{ba} - \Gamma_{bc} \sigma_{bc}, \quad (5b)$$

where $\Gamma_{ba} = \gamma_{ba} - i\delta_p$ and $\Gamma_{bc} = \gamma_{bc} - i\Delta$ are the complex decay rates for the optical coherence σ_{ba} and the Raman coherence σ_{bc} , which include the probe field detuning $\delta_p = \omega_p - \omega_{ab}$ and the two-photon detuning $\Delta = \omega_p - \omega_c - \omega_{cb}$. The vacuum Rabi frequency of the probe field is defined by $g_p = \sqrt{\frac{\omega_p}{2\hbar\epsilon_0 V}} \mathbf{e}_p \cdot \mathbf{d}_{ba}$, where \mathbf{d}_{ba} is the dipole matrix element for the $a \leftrightarrow b$ transition, and Ω_c is the Rabi frequency of the coupling field which has the decomposition

$$\Omega_c(z, t) = \Omega_c^+(t)e^{ikz} + \Omega_c^-(t)e^{-ikz}. \quad (6)$$

We have neglected the noise operators in the Heisenberg-Langevin equations (5) since it has been shown [12] that the effect of these is negligible in the adiabatic limit we shall consider here.

The Heisenberg-Langevin equations (5) can be further simplified by assuming that the coupling laser Rabi frequency and the probe field operator change slowly in time. By introducing a characteristic timescale T and expanding the Heisenberg-Langevin equations in powers of $(\gamma_{ba}T)^{-1}$, we find to lowest non-vanishing order

$$\sigma_{bc} = -\frac{g_p E_p}{\Omega_c} = -g_p \frac{E_p^+ e^{ikz} + E_p^- e^{-ikz}}{\Omega_c^+ e^{ikz} + \Omega_c^- e^{-ikz}}. \quad (7)$$

Inserting this expression into (5b) yields

$$\sigma_{ba} = \frac{-g_p (\Gamma_{bc} + \frac{\partial}{\partial t})}{i\Omega[1 + 2|\kappa^+||\kappa^-|\cos(2kz + \phi)]} \times \left(\frac{E_p^+ e^{ikz} + E_p^- e^{-ikz}}{\Omega} \right), \quad (8)$$

where we have introduced the time dependent total Rabi frequency $\Omega(t) = \sqrt{|\Omega_c^+|^2 + |\Omega_c^-|^2}$ and the Rabi frequency ratios $\kappa^\pm = \frac{\Omega_c^\pm}{\Omega}$ which are assumed to be constant. The phase angle ϕ is defined by the relation $\kappa^+ \kappa^{-*} = |\kappa^+||\kappa^-|e^{i\phi}$.

The evolution of the slowly varying probe field operators (2) in the slowly varying amplitude approximation is governed by the wave equations

$$\left(\frac{\partial}{\partial t} \pm c \frac{\partial}{\partial z} \right) E_p^\pm(z, t) = ig_p N_z \sigma_{ba}^\pm(z, t), \quad (9)$$

where the two components σ_{ba}^\pm of the optical coherence with spatial dependence $e^{\pm ikz}$ are found by Fourier expanding equation (8).

We now introduce new field operators, in analogy with the dark-state polariton field introduced in [9], defined by

$$E_p^\pm(z, t) = \cos\theta(t)\Psi^\pm(z, t), \quad (10)$$

where the angle θ is given by $\tan\theta(t) = \frac{g_p \sqrt{N_z}}{\Omega(t)}$.

Assuming that $|\kappa^+| \geq |\kappa^-|$ and considering the low group velocity limit ($\cos^2\theta \ll 1$), we obtain a set of coupled wave equations for the polariton field components

$$\left(\Gamma_{bc} + \frac{\partial}{\partial t} \right) \Psi^+ + |\kappa^+|^2 v_g \frac{\partial \Psi^+}{\partial z} = \kappa^+ \kappa^{-*} v_g \frac{\partial \Psi^-}{\partial z}, \quad (11a)$$

$$\left(\Gamma_{bc} + \frac{\partial}{\partial t} \right) \Psi^- - |\kappa^+|^2 v_g \frac{\partial \Psi^-}{\partial z} = -\kappa^{+*} \kappa^- v_g \frac{\partial \Psi^-}{\partial z}. \quad (11b)$$

We consider the type of experiment performed by Bajcsy *et al.* [19] in which a probe pulse is stored as a Raman coherence in the medium, using copropagating traveling wave lasers, and subsequently retrieved by a standing wave coupling field (see timing diagram in Fig. 1). Assuming that the standing wave coupling field is switched on at $t = 0$, the initial condition for the Raman coherence of the atoms is

$$\sqrt{N_z} \sigma_{bc}(z, 0) = -\Psi(z, 0), \quad (12)$$

where $\Psi(z, 0)$ is a known function of z found by solving the traveling wave light storage problem covered in [9]. Inserting the initial condition for the Raman coherence into (7) and using the definition of the polariton field (10), we obtain the initial conditions for the two components $\Psi^\pm(z, 0)$ of the polariton field

$$\Psi^+(z, 0) = \kappa^+ \Psi(z, 0), \quad \Psi^-(z, 0) = \kappa^- \Psi(z, 0). \quad (13)$$

With these initial conditions the solution to the wave equations (11) is

$$\Psi^+(z, t) = \frac{\kappa^+}{2} \left[\left(1 + \frac{\beta}{|\kappa^+|^2} \right) \Psi(z - \beta r(t), 0) + \left(1 - \frac{\beta}{|\kappa^+|^2} \right) \Psi(z + \beta r(t), 0) \right] e^{-\Gamma_{bc} t} \quad (14a)$$

$$\Psi^-(z, t) = \frac{\kappa^-}{2} \left(\Psi(z - \beta r(t), 0) + \Psi(z + \beta r(t), 0) \right) e^{-\Gamma_{bc} t}, \quad (14b)$$

where $\beta = \sqrt{|\kappa^+|^2(|\kappa^+|^2 - |\kappa^-|^2)}$ and

$$r(t) = \int_0^t c \cos^2 \theta(t') dt'. \quad (15)$$

In the case of a standing wave coupling field ($\kappa^+ = \kappa^- = \frac{1}{\sqrt{2}}$), the solution becomes

$$\Psi^+(z, t) = \frac{1}{\sqrt{2}} \Psi(z, 0) e^{-\Gamma_{bc} t}, \quad (16a)$$

$$\Psi^-(z, t) = \frac{1}{\sqrt{2}} \Psi(z, 0) e^{-\Gamma_{bc} t}. \quad (16b)$$

Having found the solution for the polariton field, the solution for the components of the probe field is easily obtained from equation (10).

The Raman coherence of the atoms is found from the adiabatic solution (7). By inserting the decompositions (2) and (6) of the probe and coupling fields, as well as the definition (10) of the polariton field, we get

$$\sqrt{N_z} \sigma_{bc} = -\sin \theta(t) \frac{\Psi^+(z, t) e^{ikz} + \Psi^-(z, t) e^{-ikz}}{\kappa^+ e^{ikz} + \kappa^- e^{-ikz}}. \quad (17)$$

It is clear that in the case of a standing wave coupling field ($|\kappa^+| = |\kappa^-|$), only the dc Fourier component is present,

$$\sqrt{N_z} \sigma_{bc}^{(0)} = -\sin \theta \Psi(z, 0) e^{-\Gamma_{bc} t}, \quad (18)$$

while in the more general case of a quasi-standing wave coupling field ($|\kappa^+| > |\kappa^-|$), we find by a binomial expansion of equation (17)

$$\begin{aligned} \sqrt{N_z} \sigma_{bc} = & -\frac{1}{2} \sin \theta \left(\Psi(z - \beta r, 0) + \Psi(z + \beta r, 0) \right) \\ & + \frac{\beta}{|\kappa^+|^2} [\Psi(z - \beta r, 0) - \Psi(z + \beta r, 0)] \\ & \times \sum_{n=0}^{\infty} \left(-\frac{\kappa^-}{\kappa^+} \right)^n e^{-2inkz} e^{-\Gamma_{bc} t}. \end{aligned} \quad (19)$$

We observe that σ_{bc} has Fourier components e^{-2inkz} with only positive n , becoming progressively smaller with increasing n .

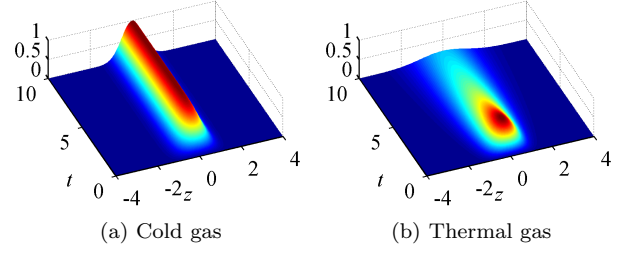


FIG. 2: (color online) Retrieval of a stored probe pulse with a standing wave coupling field. The normalized probe field energy density is plotted for a medium comprised of: (a) cold atoms, (b) thermal atoms. Time t is in units of the switching time T_s , position z is in units of the pulse length $L_p = v_{g,0} T_s$. The absorption length of the media is taken to be $l_a = 0.1 \times L_p$.

Comparing our solution (14) for the stationary atom case to the solution for the thermal gas case presented in [21], we find significant qualitative and quantitative differences. First, the diffusive broadening of the retrieved probe pulse seen in the thermal gas case is absent in a medium comprised of stationary atoms. This makes such media ideally suited for quantum information processing applications, since the dissipative losses associated with the broadening are also absent. In Fig. 2 the retrieval of a stored probe pulse by a standing wave coupling field in ultra cold atomic gasses and in thermal atomic gasses are compared. As an example, the initial conditions for the components Ψ^\pm of the polariton field are given by the function $\Psi(z, 0) = \Psi_0 \exp(-(z/L_p)^2)$ and the time dependence of the coupling field is given by $\cos^2 \theta(t) = \cos^2 \theta_0 \tanh(t/T_s)$ for $t \geq 0$, where T_s is the characteristic switching time. We have taken the length of the stored probe pulse to be $L_p = v_{g,0} T_s$, where $v_{g,0} = c \cos^2 \theta_0$ is the group velocity of the polariton field prior to storage, and assumed negligible ground state dephasing ($\Gamma_{bc} = 0$). The absorption length of both media in the absence of EIT is taken to be $l_a = 0.1 \times L_p$. In actual experiments, the dephasing rate γ_{bc} ranges from a few kHz in atomic gasses [2, 3] to a few tens of kHz in solid state media [8], allowing storage times much longer than the typical temporal length of the probe pulse, which is of the order of a few μs . The inclusion of a non-zero dephasing rate would therefore not significantly alter the results presented here.

Secondly, the behavior of a probe field retrieved by a quasi-standing wave coupling field is very different in the stationary atom case. In Fig. 3 the retrieval of a stored probe pulse by a quasi-standing wave coupling field with $\kappa^+ = \sqrt{0.55}$, $\kappa^- = \sqrt{0.45}$ in ultra cold atomic gasses and thermal atomic gasses are compared. It was shown in [21] that a probe field retrieved by a quasi-standing wave coupling field in a thermal gas medium acquires a small group velocity $v_g = (|\kappa^+|^2 - |\kappa^-|^2) c \cos^2 \theta$.

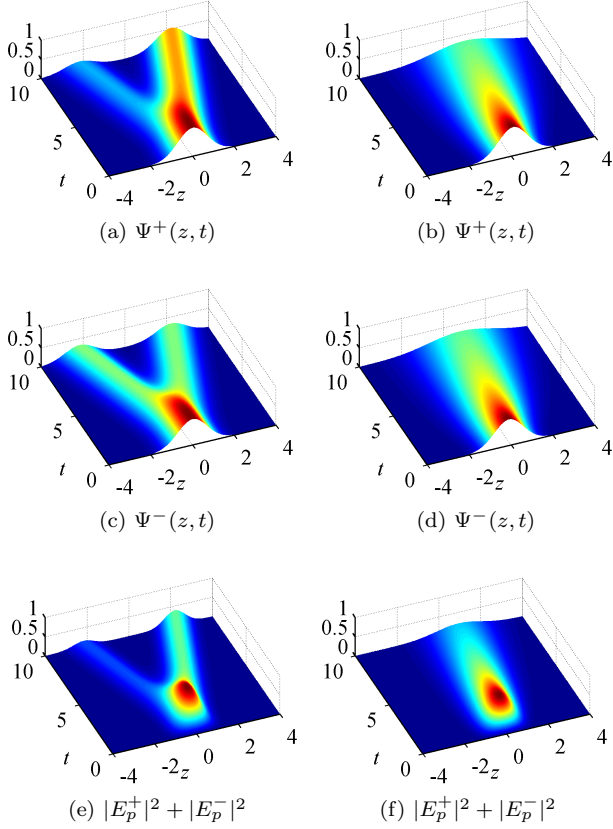


FIG. 3: (color online) Retrieval of a stored probe pulse with a quasi-standing wave coupling field ($\kappa^+ = \sqrt{0.55}$, $\kappa^- = \sqrt{0.45}$). The components of the polariton field and the probe field intensity is shown for both the ultra cold gas case (left column) and for the thermal gas case (right column).

Our solution (14) shows that in the case of stationary atoms, the revived probe field splits into two distinct parts: a stronger part propagating in the direction of the stronger component of the coupling field and a weaker part propagating in the opposite direction with velocities $v_g = \pm \sqrt{|\kappa^+|^2(|\kappa^+|^2 - |\kappa^-|^2)}c \cos^2 \theta$. We attribute the difference between moving and stationary atoms to the presence of the spatially rapidly varying Fourier components of the Raman coherence in the stationary atom case. These components undergo rapid dephasing due to the motion of the atoms, and are therefore suppressed in a thermal gas medium.

In summary, we have developed a theoretical treatment of light storage and retrieval using standing wave coupling lasers in a medium comprised of stationary atoms, such as an ultra cold gas or a solid state medium. We find that the diffusive broadening of the probe pulse seen in a thermal gas medium is absent in a medium comprised of stationary atoms, and that the behavior of a probe pulse retrieved by a quasi-standing wave coupling field is significantly different in the two types of media. These

differences have important consequences for experiments and applications of slowly propagating and stationary light. For example, the splitting, rather than slow propagation, of the probe field in a quasi-standing wave (see Fig. 3) prevents the nonlinear optical interaction proposed in [20] between a slowly propagating probe pulse and a stored polariton in media comprised of stationary atoms. Our results suggest to implement the opposite protocol with a stationary probe pulse interacting with a slowly propagating polariton in such media. The longer interaction times and stronger interactions possible in the absence of diffusive broadening (see Fig. 2) make media comprised of stationary atoms ideally suited for quantum information processing applications.

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